

GLvrb gvĭ 5uĭ `j Ges 11 Rb tLjt vqvo itqtQ etj cĭZuĭ gvbĭ mĭt_ cĭZuĭ gvbĭK Zĵbv Kĭi tKvb `tj tekx D"PZv m=ubæLjt vqvo itqtQ Zv ejv m=æ nĭ"Q| ũKŠ' cĭKZ. Mĭel Yvq AĭbK tekx Pj K Ges AskMhYKvix _vĭK etj cĭZuĭ e"i mĭt_ cĭZuĭ e"i Prĭĭĭfĭrte Zĵbv Kiv m=æ nq bv| tm tĭĭĭĭ Gkĭu Pj tKi web"vĭmi %enkó", tj vĭK Gkĭu ev `Ńjĭ mSL"v ĩ tĭ cĭKĭv Kivi cĭqRb nq| bvgmPK Pj tKi tĭĭĭĭ kZKiv, AbsvZ ev mgvbgvZ ũbYĭqi gva"tg Zv m=ubæKiv hvq Ges GB cĭi gvc, tj v ci`úi cĭi eZĭq (interchangable)| KĭvRB t`Lv hv"Q th, Gkĭu gvĭ cĭi gvc ĩ tĭB DcvĭĖi eYĭv Kiv m=æ| ũKŠ' e"ĭbgj-K Pj tKi tĭĭĭĭ cĭZubæZĭvix gvbĭ ũbYĭqi Rb" web"vĭmi gĭa" gvb, tj vi Ae"ĭbĭK chĭeĭY KĭtZ nq| GB Ae"ĭb ũbYĭqKivix gvbĭ ũbYĭq Rb" web"vĭmi Ab"vb" gvb, tj v, "Ńex _vĭK| Gĭu ũbYĭq Gkĭu tK>"ĭq gvb Ges MYmsL"v web"vĭmi Ab"Zg cĭvb "enkó"|

Pj tKi web"vĭmi %enkó", tj vĭK Gkĭu ev `Ńjĭ mSL"v ĩ tĭ cĭKĭv Kivi cĭqRb nq| bvgmPK Pj tKi tĭĭĭĭ kZKiv, AbsvZ ev mgvbgvZ ũbYĭqi gva"tg Zv m=ubæKiv hvq|

tK>"ĭq cĕYZv ĩK (What is Central Tendency)?

th tKvb ũbtektĭi Ašĭfŷ ĩmK Mĭj v ũeĭfbægvĭbi nĭq _vĭK| tKvb tKvb gvb AĭaKevĭ AĭeĭfZ nq Ges tKvb gvb GK ev `Ńevi-Gi tekx cĭi j ĩĭZ nq bv| Gkĭu web"vĭmi mgĭoMZ %enkó" ũbaĭĭĭY GB `ŃĭiĭYi gvbB, ĩæZĭYĭ Kiv Y, ũbtektĭi gĭa" gvbĭ ũeĭfbæAe"ĭb D³ ũbtektĭK eYĭvi Rb" Gkĭu GKK mSL"v ũbYĭq, ĩæZĭYĭfĭgKv cĭj b Kĭi| GB cĭv Avgiv ũbtektĭi gvb, tj vi NbrfZ Ae"ĭbĭi ũelqĭi cĭZB Avgĭĭ ĩ`Ń ũbex ĩvLĭev| tKvb ũbtektĭK chĭeĭY Kĭtj t`Lv hvq th, AĭaKĭsk gvbB Gkĭu gvbĭK tK>"Kĭi NbrfZ nĭq itqtQ| tKvb ũeĭKĭ gvbĭK tK>"Kĭi AĭaKĭsk gvbĭ cĭĭfZ nevi GB cĕYZvĭK tK>"ĭq cĕYZv etj| ũKŠ' GB gvbĭ cĭevi tKvb Gkĭu Abb" mĭ (unique formula) tĭB| Gkĭu MYmsL"v ũbtektĭi Ae"ĭbĭK msĭvĭqZ Kivi Rb" ũeĭfbæ cĭKĭv cĭi gvc ĩtqtQ| GB cĭi gvc, tj vĭK tK>"ĭq cĕYZvĭ cĭi gvc etj| A_ĭ, tK>"ĭq cĕYZvĭ cĭi gvc ej tZ Avgiv Ggb Gkĭu mSL"vĭK eĭŷ hv Gkĭu ũbtektĭi Ašĭfŷ mKj gvbĭK cĭZubæZĭKĭi|

tKvb ũeĭKĭ gvbĭK tK>"Kĭi AĭaKĭsk gvbĭ cĭĭfZ nevi cĕYZvĭK tK>"ĭq cĕYZv etj| tK>"ĭq cĕYZvĭ cĭi gvc ej tZ Avgiv Ggb Gkĭu mSL"vĭK eĭŷ hv Gkĭu ũbtektĭi Ašĭfŷ mKj gvbĭK cĭZubæZĭKĭi|

Mo – tK>"ĭq cĕYZvĭ cĭi gvc (Averages – Measures of Central Tendency)

Avgiv tRĭbĭQ th, Gkĭu ũbtektĭi Ašĭfŷ DcvĭĖiĭki gvb ũeĭfbæĭtjĭ AĭaKĭsk gvbB ũbtektĭi Gkĭu ĩvĭb tK>"ĭfZ nevi cĕYZv cĭkĭ Kĭi| Avgiv Avĭĭ tRĭbĭQ th, ũbtektĭi Gkĭu ũeĭKĭ ĩvĭb DcvĭĖiĭki tK>"ĭfĕbĭK Gkĭu msĭĭĭB ev tK>"ĭq cĭi gvcgĭ-K gĭtĭ cĭKĭv Kivi tKvb Gkĭu Abb" mĭ tĭB, eis Gi ũeĭfbæcĭi gvc ĩtqtQ| GB cĭi gvc, tj v ũeĭfbæbvĭtg cĭi ĩPZ| thgb, Mo, Ae"ĭbĭi cĭi gvc, Ges tK>"ĭq cĕYZvĭ cĭi gvc, BZ"ĭ | th bĭtgB cĭi ĩPZ tĭvK bv tKb, G, tj vi AšĭbĭZ A_ĭ Dĭĭĭ" GK Ges Aĭfĭbĭ

mĭvĭ Yĭfĭte Mĭoi avĭYvi mĭt_ Avgiv meĭB cĭi ĩPZ Ges cĭZĭ"b Avgiv ũeĭfbĭfĭte Gi e"envĭ Kĭi _ĭĭK| thgb, ŃgvSĭmĭ aiĭYi QvĭŃŃ, ŃMo ZvcgvĭŃŃ, Ńga"meĖ tKĭYĭŃ, ŃMo AvqŃ, BZ"ĭ | ĩvĭĭ Mĭoi G aiĭYi Aĭfe"ĭ³ Gkĭu Av"ĭŃ cĭZubæZĭĭj gvbĭK ũb"ĭ Kĭi, ũKŠ' cĭi gvcĭi cĭKĭZ teĭSvi Rb" Zv "úo bq| tK>"ĭq cĕYZvĭ cĭi gvc ĩmĭte GB c"ĭi mĭbĭŷ Ges h_v_ĭA_ĭĭitqtQ hv mĭvĭ Y gvbĭĭi tMĭPĭxfZ bq| cĭKZcĭĭ, ũeĭfbæcĭĭvĭi Mo ĩtqtQ hv GK Gkĭu web"ĭm m=úĭK"mĭgZ Ges ũfbæavi Yv ĩ tĭ _vĭK| GB ũfbæv KLbĭ KLbĭ ũfbædjĭĭ ĩ tĭ _vĭK| GRb" Avgĭĭ ĩ ũeĭfbæcĭĭvĭ Mo m=úĭK"Rĭvĭ Ges tm, tj vi gĭa" cĭ_ĭ", tj v teĭSv cĭqRb| tKvb cĭi ĩvĭZĭZ tKvb aiĭYi Mowĭ cĭĭvĭR", tm, tj vi ũbYĭ cĭĭZ ĩK Ges cĭĭ djĭĭtj ĩ e"ĭL"v ĩK, Avgĭĭ ĩ Zĭĭ Rĭvĭ cĭqRb| GB ũelq, tj v m=úĭK" mg"K avĭ Yv bv ũbĭtj tK>"ĭq cĕYZvĭ cĭi gvc, tj vĭK mĭvĭfĭte Abĭĭeb Kiv m=æ bq|

tKvb cĭi ĩvĭZĭZ tKvb aiĭYi Mowĭ cĭĭvĭR", tm, tj vi ũbYĭ cĭĭZ ĩK Ges cĭĭ djĭĭtj ĩ e"ĭL"v ĩK, Avgĭĭ ĩ Zĭĭ Rĭvĭ cĭqRb|

Mo wbyqji Df'ik" (Objectives of Computing Averages)

Mo wbyqji "gu c'vb Df'ik" itqtQ:

Mo wbyqji "gu c'vb
Df'ik" itqtQ: c'gZt,
GKuU GKK I msu'fB
cwi gvcgj-K msL'v gvb
cvi qv| wZxqZt, Z'bv'K
mnRZi Kiv|

c'gZt, GKuU GKK I msu'fB cwi gvcgj-K msL'v gvb cvi qv, hv c'jiv Z'wki 'ewk'fK eY'v Kfi | thgb, GKuU Rb'tMv'xi mKj m'tm'i uk'f'vMZ thM'Zv gtb ivLv m'e bq Ges Gi c'qvrBl tbB| G ai'Yi GKuU Ab'xj b nte A'xj| Kvi Y, c'ZuU gvb tMv'x ev mgu'oi Ask n'tj I GuU'K c'ZubwaZi; Kfi bv| mK's'hw' Argiv GKuU Rb'tMv'xi Mo uk'f'vMZ thM'Zv Rvb'tZ cwi, Zintj Argiv GKuU GKK msL'v cvB hv c'jiv Rb'tMv'xi GKuU mgu'omZ 'ewk'fB i'ayeY'v Kfi bv, GuU'K c'ZubwaZi Kfi |

wZxqZt, Z'bv'K mnRZi Kiv| Dcv'E msu'fBK i'Yi gva'tg th tK'x'g gvbwU cvi qv hvq Zv Avgv' i G'Ki Awak web'v'tmi g'ta' Z'bv Ki'tZ min'v'h' Kfi | i'ayZvB bq, tK'x'g gvb 'vri GKuU web'v'tmi A's'f' mKj g'v'tbi m'v' I Z'bv Kiv m'e nq| GK K'vq, Mo 'vb, Kij I cvl' t'f' Z'bv'K mnRZi Kfi t'v'tj |

tK'x'g c'eYZvi cwi gvcmg' (Measures of Central Tendency)

tK'x'g c'eYZvi we'f'ba'cwi gvc' t'j v n'tj v:

- c'p'zK (Mode)
- ga'gv (Median)
- M'w'v'ZK Mo (Arithmetic Mean)
- R'w'v'ZK Mo (Geometric Mean)
- Zi 1/2 Mo (Harmonic Mean)

mgv'R M'tel Yvq c'vbZt
wZbuU M'toi e'envi f'Lv
hvq - c'p'zK, ga'gv Ges
M'w'v'ZK Mo|

mgv'R M'tel Yvq c'vbZt c'g wZbuU M'toi e'envi f'Lv hvq| cieZ'p'cv'wmg'f' GB wZbuU M'toi we'v'v'Z Av'tj vP'v Kiv n'te| mij Zv I i'at'Zi g'v'v Ab'jv'q' c'g c'p'zK, Zv'cti ga'gv Ges me t'k'l M'w'v'ZK Mo'K Av'tj vP'v Kiv n'te| Z'te c'ZuU M'toi we'v'v'Z Av'tj vP'v'v c'te'f'K'x'g c'eYZvi GKuU D'Eg cwi g'v'tci uk' uk' 'ewk'f' v'Kv c'qvrB Avgiv Zv D'tj L K'it'v|

tK'x'g c'eYZvi GKuU D'Eg cwi g'v'tci Kuw'LZ 'ewk'f' mg' (Desirable Characteristics of a Good Measure of Central Tendency)

tK'x'g c'eYZvi cwi gvcmg' th'tnZzGKuU web'v'tmi A's'f' mKj gvb'tK GKuU GKK msL'v 'vri c'ZubwaZi; Kfi, t'm'tnZzGKuU cwi g'v'tK D'Eg e'tj we'te'v'PZ n'tZ n'tj t'mB cwi g'v'tci g'ta' wbg'j'w'LZ 'ewk'f' mg' v'Kv ev'v'v'q|

c'gZt, M'toi GKuU m'y'uo ms'Av v'K'tZ n'te, hv'tZ Kfi GuU GKuUB e'v'v'v' f' I qv m'e nq| m'v'v'v' ms'Av bv v'K'tj cwi g'v'v'v'K f'j'v tev'Svi Ae'K'v' v'tK Ges GuU'Z c'q'cv'Zg'j-K a'v'v'v' m'e'v'v' v'tK| Av'tiv m'y'uo'v'v'te ejv hvq th, M'toi ms'AvU GKuU ex'R'M'w'v'ZK m'f'i g'v'v'v' c'K'v'v' Kiv ev'v'v'v'q, hv'tZ Kfi GKB i'w'kg'j'v e'envi Kfi ev'v'v'v' GKB dj'v'dj cvi qv hvq|

wZxqZt, GuU mnR'teva' n'tZ n'te| Zv bv n'tj c'vB cwi gvcgj-K msL'v web'v'tmi mgu'omZ 'ewk'f' c'K'v'v'v' Df'ik'f'K e'nZ K'it'e|

cW - 2

**cPžK
Mode**

GB cvW tkłI hv Rvbr hrte —

- cPžK wK
- web`IDcvE t_žK cPžK wbyq c×wZ
- cti vŋ Dcvŋq cPžK wbyq c×wZ
- GžKi Awak cPžK
- tj LwPžI gva`tg cPžK wbyq
- cPžžKi mjeav I Amjeav

cPžK wK (What is Mode)?

cPžK nŋv tk`iq cEYZvi meŋPŋq mnRZi cwi gvc | DcvE mšMŋni ci hLb žm, tj vŋK GKw MYmsL`v wŋekžb mvrivŋv nq, ZLb t`Lv hvq th, wKQymsL`v gvb žKvb žKvb tkYŋZ Awakvi AwefZ nŋqŋ | meŋak msL`v gvb mŋj Z tmB tkYx ev tkYx, tj vŋK cPžK tkYx ej v nq |

cPžK kawi gŋa`B Gi A_ŋbwZ i žŋqŋ | hv cPž cwi gvŋY NŋU tmiUB cPžK | A_ŋ, GKw web`vŋmi gŋa` meŋakvi th gvbw NŋUŋQ tmB gvbwžK cPžK etj | wŋtge`kw msL`vi GKw Z_`i wK Dc`rcb Kiv nŋqŋQ,

30 45 30 20 25 30 45 20 30 30

hv cPž cwi gvŋY NŋU tmiUB cPžK | A_ŋ, GKw web`vŋmi gŋa` meŋakvi th gvbw NŋUŋQ tmB gvbwžK cPžK etj |

GB Z_`i wKžK chŋŋY Kiŋ j`ŋ` Kiv hvq th, 30 msL`w mveŋakvi NŋUŋQ | GB 30 msL`wUB GB DcvE wki cPžK | wekpe`ij q K`vŋdŋw qvq Dcw`Z Qvŋ-Qvŋ` i gŋa` Rwi c Kŋi t`Lv tMj th, Zv`i gŋa` 12 Rb cŋg etIŋ, 23 Rb wZxq etIŋ, 18 Rb ZZxq etIŋ Ges 15 Rb PZžetIŋ Qvŋ-Qvŋx i žŋqŋ | GLvŋb cPžK nŋv wZxq etIŋ Kvi Y, wZxq etIŋ Qvŋ-Qvŋ` i msL`v meŋPŋq tekx cvl qv wŋtŋqŋ | Avŋi Kw D`vni Y t`qv hvK | GKw D`P Rbŋvi mŋj Z Rbŋvŋx cwi Pwŋj Z GKw Mŋel Yvq 200 Rb bvi xi cPžžK KqW mšŋb Rbŋv žŋqŋ Zvi web`vmi mvi wY 4.2.1-G Dc`wcz nŋj v:

mvi wY 4.1.1: 200 Rb bvi xi mšŋb msL`vi MYmsL`v web`w

mšŋb msL`v	1	2	3	4	5	6	7	8	9
bvi xi msL`v	10	16	29	36	47	29	18	11	4

Dcŋi web`vmiŋZ t`Lv hvŋ`Q th, 5w mšŋb Rbŋv žŋqŋQb meŋak msL`K bvi x (47 Rb) | GLvŋb 5 nŋ`Q GB web`vŋmi cPžK mšŋb msL`v | GB D`vni Y, tj v t_žK tk`iq cEYZvi cwi gvc wmwte cPžžKi cāvŋ, Yw cKwKZ nq th, th žKvb gvŋvq cwi gvcKZ. DcvE t_žK cPžK wbyq Kiv mŋe |

web`-l DcvE t_tK cPzK wBYq (Computing Mode from Grouped Data)

mij Ges Kg msL`K bgbvi t`q`l cPjK wBYq wZvSB mnR| wKs`hLb tkYxex MYmsL`v wtekb t_tK (thLvtb cPzK GKwU tkYxi gta` Ae`vb Kti) cPzKtK wBYq Kitz nq, ZLb welqW GZ mnR_vtK bv| KviY, MYmsL`v wtekb t_tK mriY MYvri gva`tg wbfv`fite GKwU h_v`cPzK wBYq Kiv m`e bq|

hw`l GKwU MYmsL`v wtektbi gta` Awak NUbmsL`v m`nj Z tkYwUB cPzK tkYx etj wPwYZ nq, wKs`cPzK tkYxi ga` t_tK cPzKti h_v`gubwU wBYq mgm`v `Zix Kti | Avgiv hw` aji tbB th, cPzK-ce`Ges cPzK-cieZPtkYxi gta` NUbmsL`v, tjv mgvbfite web`-l itqtQ| tmB t`q`l cPzK tkYxi ga`-w`w`cPzK etj MY` Kiv hvq| wKs`l ev`te NUbmsL`v, tjv mgvbfite web`-l_vtK bv etj wtkl ai`Yi m`F cPqv`Mi gva`tg tmB Amg web`v`mi cFv`tK gy` Kti wtz nq| cPzK wBYqi tmB wtkl m`FwU ntjv,

$$cPzK = L_1 + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C.I.$$

$$L_1 = cPzK \text{ tkYxi wbgomxgv}$$

$$\Delta_1 = cPzK \text{ l cPzK-ce`tkYxi NUbmsL`vi gta` cv`R`}$$

$$\Delta_2 = cPzK \text{ l cPzK-cieZPtkYxi NUbmsL`vi gta` cv`R`}$$

$$C.I. = \text{tkYx e`wB}$$

m`FwU`K Avti Kfite Dc`vcb Kiv hvq,

$$cPzK = L_1 + \left(\frac{(f_0 - f_1)}{(f_0 - f_1) + (f_0 - f_2)} \right) \times C.I.$$

thLvtb,

$$L_1 = cPzK \text{ tkYxi wbgomxgv}$$

$$f_0 = cPzK \text{ tkYxi NUbmsL`v}$$

$$f_1 = cPzK-ce`tkYxi NUbmsL`v$$

$$f_2 = cPzK-cieZPtkYxi NUbmsL`v$$

$$C.I. = \text{tkYx e`wB}$$

mvi wY 4.2.2-G cD`E DcvE e`envi Kti cPzK wBYqi GKwU D`vni Y t`qv nj |

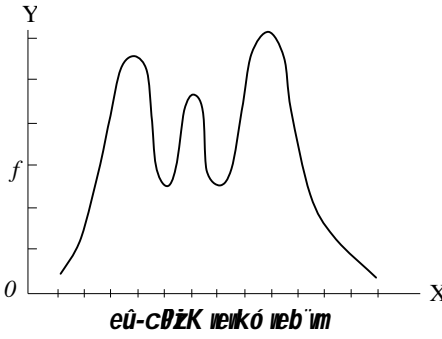
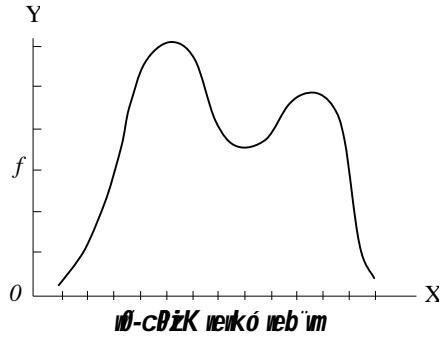
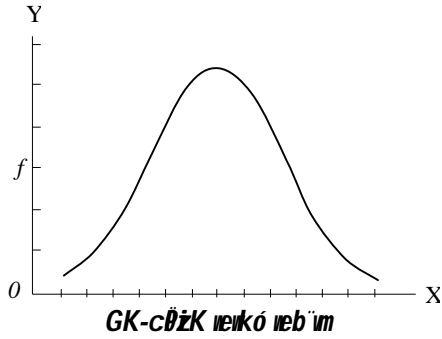
mvi wY 4.2.2: GKwU wkte`vj tqi wvfbw`fvtMi QvT`-QvTx msL`vi web`m

tkYx mgv	NUbmsL`v
0 – 50	25
50 – 100	50
100 – 150	90
150 – 200	30
200 – 250	26
250 – 300	39

GĩKi AwaK cPžK (More than One Mode)

DcvEiviki GKwU vřb NbrfZ nI qv web'ımřK GK-cPžK weikó web'ım etj, ğU vřb NbrfZ nI qv web'ımřK w-cPžK weikó web'ım Ges Ğ-Gi AwaK vřb NbrfZ nI qv web'ımřK eü-cPžK weikó web'ım etj |

G chšI Avgiv th mKj D`vniY wbtq Avřj vPbv KřiuQ tm,tjv meB GK-cPžK weikó (unimodal) web'ım | wKš' AřbK mgq DcvEiviki GKwU Ae vřb NbrfZ bv nřq Ğ ev ZřZwaK vřb řKřřfZ vřK | G řřřř Dcřiv³ mř e`envi Kři GKwU cřZwbaZřřj cPžK wbyř Kiv mře bq | DcvEiviki GKwU vřb NbrfZ nI qv web'ımřK GK-cPžK weikó (unimodal) web'ım etj, ğU vřb NbrfZ nI qv web'ımřK w-cPžK weikó (bimodal) web'ım Ges Ğ-Gi AwaK vřb NbrfZ nI qv web'ımřK eü-cPžK weikó (multimodal) web'ım etj | DcvEiviki cřwZ. I `eikó, AřřřřřK řkřř web'ım, ř ř cwi qv řkřř e`wř, Ges ř ř mšL`K bgřvi KviřY G aiřYi GKwaK cPžK weikó web'ımři mřřřř nřZ nq | hw GKwU web'ımř GĩKi AwaK cPžK vřK Zře meřřř tekř NUbmsL`v mřřř Z cPžKwřK cřwřK cPžK (primary mode) Ges Ab`vb` cPžK,tjřK řMSY cPžK (secondary mode) etj |

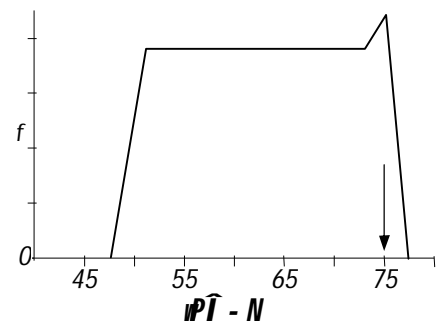
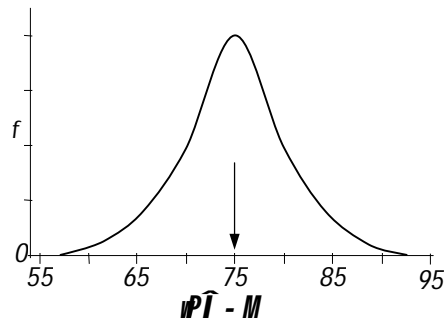
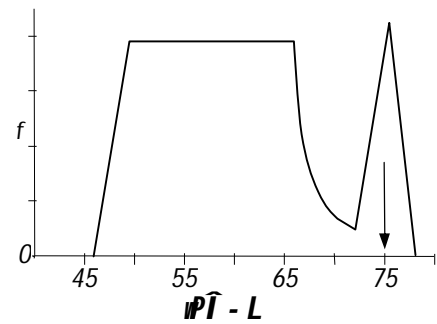
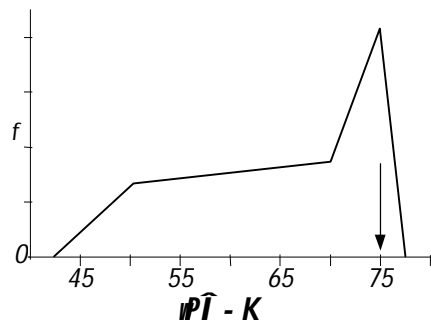


řřř 4.2.1: weřřřcPžK weikó web'ım

eü-cPžtKi Dcw~wZi
 Kvi tY KLbl KLbl
 cPžtK msAwqZ Kiv mæ
 nq bvl hw GKwU web'vfm
 chfeŋY msL'v ht_ó
 cwi gvtY tekx bvl nq Ges
 chfeŋY t'v GKwU ~úó
 "QexZv cõkõ bvl Kti, tm
 t'ŋt' cPžtKi Ae~vb wYq
 Kiv tek Kõmva" ntq cto|

mæavi cvkvcvk cPžtKi wKQyAmæavl itqtQ| eü-cPžtKi Dcw~wZi Kvi tY KLbl KLbl
 cPžtK msAwqZ Kiv mæ nq bvl hw GKwU web'vfm chfeŋY msL'v ht_ó cwi gvtY tekx
 bvl nq Ges chfeŋY t'v GKwU ~úó "QexZv cõkõ bvl Kti, tm t'ŋt' cPžtKi Ae~vb
 wYq Kiv tek Kõmva" ntq cto| exRMwYwZK e'envi i Rb" cPžK DcthvMx bq| Kvi Y,
 GwU web'vfm mKj gvbtk e'envi Kti bvl etj Gi tKvb thvMdj cvl qv hvq bvl dtj, `B ev
 ZtZwAK cPžtKi mæšj Z cPžKl wYq Kiv hvq bvl cPžK wYq kõmva" Kvi Y, cPžK
 wYq cte©DcvEivntK mivrtZ nq, tkYæx KitZ nq, Ggb wK, KLbl KLbl
 cPžtkYæxKiYl KitZ nq|

~f msL'K DcvEi t'ŋt' hw Kgcŋŋ ŋwU chfeŋY GKB iKg bvl nq, tm t'ŋt' cPžK
 wYq Kiv hvq bvl Kvi Y, tmLvtb tKvb cPžK gvb ~vtK bvl cPžK bgbv wPžZ ōviv tekx
 cŋwæZ nq| DcvEi tkYæxKiY cPžtKi Ae~vbtk cŋwæZ Kti | cPžtK Av`kŋK`ŋq
 gvb wnmvte eYŋvi t'ŋt' metPŋ cãvb AmæavwU ntj v th, GwU cPžK tkYæ euvfZ NUBmsL'v
 ōviv cŋwæZ nq bvl etj wKQyMQt'ŋt' Gi e'envi GKwU web'vm mæútK©ãvšl'ari Yv cõvb
 Kti | cPžK wKfvte GKwU web'vm mæútK©ãvšl'ari Yv w' tZ cvti Zv wPŋ 4.2.2K, L, M Ges
 N-G PviwU web'vfm PviwU tj LwPŋi gva'tg t' Lvtbv ntj v|



pŋ 4.2.3: GKB cPžK gvb wnkó PviwU web'vfm gva'tg cPžtKi mææxZv cõkõ

pŋ K, L, M Ges N-G t'Lv hvq th, cãZwU t'ŋt' cPžtKi gvb GK – 75, A_P cãZwU
 web'vfm cKwZi gta" e'vcK cv`R" itqtQ| GB wfbwckwZi web'vfm cPžK gvb GK ntj l
 tmB gvbwU wKQtZB web'vm t'v Ašfŋ chfeŋYmgfni cãZwvwaZi Kti bvl ZvB cPžK,
 G t'ŋt' tK`ŋq cãYZvi GKwU ãvšlcwi gvc wnmvte cwi MwYZ nte|

mvi usk

cřžK nřjv řK`řq cřYZvi meřPřq mnRZi cwi gvc | GKwU webřřm hv cřž cwi gvrřY NřU
řmiUB cřžK | DciřEi vřki GKwU řřb NbrřZ nř qv webřřmřK GK-cřžK weřkó (unimodal)
webřřm eřj, `řU řřb NbrřZ nř qv webřřmřK wř-cřžK weřkó (bimodal) webřřm Ges
`ř-Gi AwaK řřb NbrřZ nř qv webřřmřK eř-cřžK weřkó (multimodal) webřřm eřj |
cřžřKi řhgb wřQzmeav iřqřQ řZgb wřQzAmřeav iřqřQ | řhgb, bvgmPK Pj řKi řřřř
cřžK nřjv GKgvř cwi gvc hv DciřEi řK`řq gvb wřř`R Kři | Aveři řKvb webřřm eř-
cřžřKi DciřřZi Kvi řY KLbř KLbř cřžKřK msřvřqZ Ki v mře nq bv |

thñZzDcivÉi mki tgvU msL'v civÉ A_ŕ, tetRvo msL'v, tmñZzG tñtÍ ga'gv wbyŕ tek mnR| msÁv Ablyqx gvSLvbi msL'vU ntjv ga'gv, A_ŕ 72| mKŠ' hw` AvZwi 3 GKRb e'w3i I Rb GB DcivÉ i mkgvj vi mŕ_ thvM Kiv hvq, ZLbB ga'gv GKwU A`úó jfc civ Mñ Kti | KviY, tmLvfb Avi tKvb ga'-gub _vŕK bv| aiv hvK, AvZwi 3 e'w3i I Rb ntjv 62 civDÚ| tmB 6ô gvbiU hŕ Kti gvŕbi Eaŕŕg Ablyvŕi mŕRvŕj DcivÉ i mkgvj wU `vovq wbgŕfc:

57 62 69 72 81 86

GLvfb tKvb ga'-gub bv _vKvq ga'gvU gvSLvbi `ŕU gvb, A_ŕ, 69 I 72 Gi th tKvb `vfb Ae`vb Kti | wvŕZŕvte ga'gv tctZ ntj msL'v `ŕUi Mo wbyŕ Kitz nte| A_ŕ, ga'gv ntjv $\frac{(69+72)}{2}$, ev 70.5 civDÚ| GwU GKwU t`QvPwqZ (arbitrary) Avwi Z (derived) tK`ŕq gvb, cKZ.ga'gv bq|

Avb`ÍDcivÉ t_ŕK ga'gv wbyŕ (Calculating Median from Ungrouped Data)

Avb`ÍDcivÉ t_ŕK ga'gv wbyŕi `ŕU mŕ i tŕtQ| hLb DcivÉi mki tgvU msL'v tetRvo nq ZLb ga'gv wbyŕi mŕU ntjv,

$$ga'gv = \frac{n+1}{2} Zg msL'v$$

aiv hvK, GKwU civŕvq 15 Rb Qvŕ-Qvŕx AskMñY Kti tQ| civŕvi djvdj Ablyqx Zŕ`i cŕB bŕt_ŕjv ntjv,

64 55 40 50 80 70 60 54 58 45 30 35 70 61 75

gvŕbi D`Pmŕg Ablyvŕi mŕRvŕj i mkgvj vi mŕU nq Gi Kg,

30 35 40 45 50 54 55 58 60 61 64 70 70 75 80

thñZztgvU Qvŕ-Qvŕxi msL'v 15, tmñZzmŕ Ablyqx $\frac{15+1}{2} Zg msL'vU$, ev 8g msL'vU ntjv ga'gv| A_ŕ, 58 nj GB i mkgvj vi ga'gv| hLb tgvU msL'vU tRvo msL'v nq, ZLb ga'gv ntjv $\frac{n}{2} Zg Ges \frac{n}{2} + 1 Zg gvŕbi Mo|$ aiv hvK, cieZŕcivŕvq AskMñY Kti tQ 12 Rb Qvŕ-Qvŕx| civŕvi djvdj Ablyqx Zŕ`i cŕB bŕt_ŕjv (gvŕbi D`Pmŕg Ablyqx) ntjv,

35 40 45 50 55 58 60 64 70 75 78 80 85

msÁv Ges mŕ Ablyqx ga'gv ntjv,

$$ga'gv = \frac{\frac{n}{2} Zg i m k + \left(\frac{n}{2} + 1\right) Zg i m k}{2}$$

$$\begin{aligned}
 &= \frac{\frac{12}{2} Zg\ i\ vnk + \left(\frac{12}{2} + 1\right) Zg\ i\ vnk}{2} \\
 &= \frac{6\hat{o}\ i\ vnk + 7g\ i\ vnk}{2} = \frac{58 + 60}{2} = \frac{118}{2} \\
 &= 59
 \end{aligned}$$

∴ ubYqZ ga`gv ntj v 59

web`I DcvE t`K ga`gv ubYq (Computing of Median from Grouped Data)

~f msL`K DcvE i vnkgyj vi t`qj t` DcvE i vnk t`K web`I b v K t i B g v t b i μ g A b h v q x m m i R t q ga`gv ubYq Kiv nq | mKŠ' mvgmRK M t e l Y v q m v a i Y Z t M t e l K t K A m a K m s L ` K D c v E i v n k u b t q K v R K i t z n q e t j D c v E i v n k g y j v t K w e b ` I K t i t b e v i c l q v R b c t o | G B w e b ` K i Y c l q v q D c v E i v n k g y j v A v c b v A v c u b w e f b a e t k Y x t z μ g A b h v q x w e b ` I n t q h v q | w e b ` I D c v E i t`qj t` c l z u l t k Y x i N U b m s L ` v μ g t h w R Z f i t e t h v M K t i t m B t h v M d j t K ` B f i t M f i M K t i g a ` g v i A e ` v b t K v b t k Y x t z c l t g Z v u b Y q K t i u b t z n q | A _ v e , t h t k Y x t z μ g t h w R Z N U b m s L ` v $\frac{n}{2}$ g v b u i m g v b e v t e k x , A _ e v t h t k Y x i μ g t h w R Z N U b m s L ` v $\frac{n}{2}$ g v b u i m g v b e v K g t m B t k Y x i g t a ` g a ` g v i A e ` v b a t i t b q v n q | t K v b t k Y x t z g a ` g v i A e ` v b Z v u b i f c t Y i c i u b t g u c l E m f e ` e n v i K t i g a ` g v u b Y q K i t z n q |

~f msL`K DcvE i vnkgyj vi t`qj t` DcvE i vnk t`K web`I b v K t i B g v t b i μ g A b h v q x m m i R t q ga`gv ubYq Kiv nq |

$$\text{ga`gv} = L_1 + \frac{\frac{n}{2} - F}{f} \times \text{C.I.}$$

- L₁ = ga`gv m`nj Z tkYxi ubg`mxgv
- F = ga`gv m`nj Z tkYxi ubg`mxgv μgthwRZ MYmsL`v
- f = ga`gv m`nj Z tkYx e`vBi Ašf` MYmsL`v
- n = t`gvU MYmsL`v
- C.I. = ga`gv m`nj Z tkYxi e`vB

mvi wY 4.3.1-G c l E D c v E e ` e n v i K t i g a ` g v u b Y q i G K u D ` v n i Y t ` q v n j |

mvi wY 4.3.1: GK u D nekje`ij t`qi w e f b a e f i t M i Q v l - Q v l x m s L ` v i w e b ` v m |

tkYx mxgv	NubmsL`v	μgthwRZ MYmsL`v
0 – 50	25	25
50 – 100	50	75
100 – 150	90	165
150 – 200	30	195
200 – 250	26	221
250 – 300	39	260

$$Dctiv^3 \text{ web'im Abjvqx, } \frac{n}{2} = \frac{260}{2} = 130Zg \text{ msL'wJB nte ga'gv}$$

AZGe, ga'gvi Ae'vbiw ntv, 75 Ges 165-Gi gta' | msikw tkYx mvgwU ntv 100–150 Ges GB mvgvi Ašfš NUBmsL'wU ntv 90 | cšB gvb, tv mfi gta' cšqvm Ki tv Argiv ga'gvi gvbU cter |

$$\begin{aligned} \text{ga'gv} &= L_1 + \frac{\frac{n}{2} - F}{f} \times C.I. \\ &= 100 + \frac{130 - 75}{90} \times 50 = 100 + \frac{55}{90} \times 50 = 100 + \frac{2750}{90} \\ &= 100 + 30.56 \\ &= 130.56 \end{aligned}$$

∴ wYkZ ga'gv ntv 130.56

we'ObwPj tKi ga'gv wYq (Computing Median from Discrete Variable)

web' DcvEimk t_tK ga'gv wYq i t'q' Avgiv j q' KtiwQ th, DcvEimk GKwU Awew'Qbre gv'vq mvrvtbv i t'q'Q | wKš' AšbK mgq DcvEimk we'ObwPj vq web' t_vtK | tm t'q' ga'gv wYq Ki tv ntv cšqg $\frac{n}{2}$ Zg msL'vi Ae'vbi wbjfcY Kti wbtZ nq | wbjfcZ ga'vbiw th $\mu gthwRZ$ gvtbi we'v'Z v_tK tmB gvtK ga'gv etj aiv nq | thgb, Argiv Rwb th, GK GKwU cwi ev'ti AvKvi GtKK iKg ntv v_tK | G iKg 100wU cwi ev'ti Rixc Kti Rvbr tmj th, cwi ev'ti m`m' msL'v 2 Rb t_tK 7 Rb chš' Qwotq i t'q'Q | GLb tKvb cwi ev'ti AvKviwU ga'gv tmwU Rvbr cšqvRb | Zv RvbtZ ntv mviw 4.3.2-G cšE DcvEimki $\mu gthwRZ$ MYmsL'v wYq Kti $\frac{n}{2}$ Zg i wki gvb wYq Ki tv nte | Dctiv^3 DcvE Abjvqx $\mu gthwRZ$ MYmsL'v wYq Ki tv web'v wU `vavq wbgwfc,

mviw 4.3.2: 100 cwi ev'ti AvKvi I cwi ev'i msL'vi web'im

cwi ev'ti AvKvi (m`m' msL'v)	cwi ev'ti msL'v	$\mu gthwRZ$ cwi ev'ti msL'v
2	10	10
3	20	30
4	30	60
5	25	85
6	10	95
7	5	100

μgthwRZ MYmsL`v ubYq̄i ci ga`gv m̄aj Z i vki gvb ntj v,

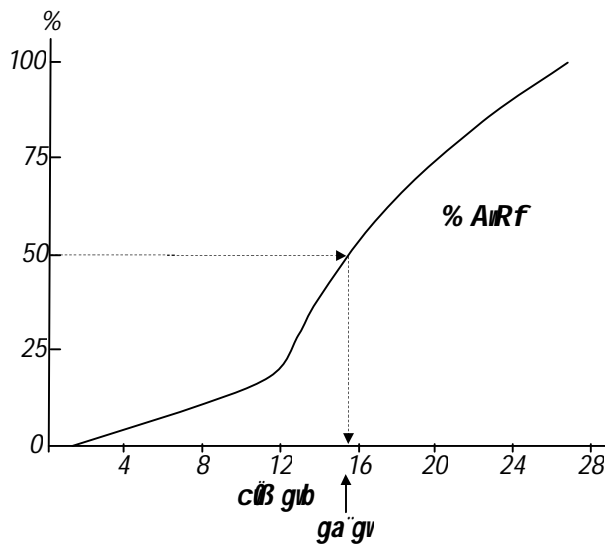
$$\begin{aligned} \text{ga}^{\text{gv}} &= \frac{n}{2} Zg \text{ i vki gvb} \\ &= \frac{100}{2} Zg \text{ i vki} \\ &= 50Zg \text{ i vki} \end{aligned}$$

μgthwRZ MYmsL`vi t̄c̄j q̄i Kiv hvq th, 50Zg i vki u i Ae`vb 60-Gi gta` Ašf̄q̄ | Kiv RB 60-Gi neci x̄Z 4 m`m` nekó cwi ev̄ti i AvKvi u B ntj v ga`gv cwi ev̄ti i AvKvi |

μgthwRZ MYmsL`v
tj LuP̄t̄i Dc`rcb Kitj th
u P̄t̄i u cvl qv hvq Zv̄t̄K
Aur̄f̄ etj | Aur̄f̄ t̄K
e`envi Kti c̄q̄ct̄Yi
gva`tg ga`gv ubYq̄ Kiv
hvq |

c̄q̄ct̄Yi ev tj LuP̄t̄i i gva`tg ga`gv ubYq̄ (Locating Median through Interpolation or Graphs)

c̄q̄ct̄Yi gva`tg ga`gv ubYq̄ Kiv hvq | Avgiv Rmb th, ga`gv ubYq̄i Rb` c̄Z̄u u NubmsL`v̄t̄K cieZ̄msL`vi m̄t̄ thw̄ Kti μgthwRZ MYmsL`v ubYq̄ Ki t̄Z nq | Avgiv GI t̄R̄t̄b̄u th, GB μgthwRZ MYmsL`v tj LuP̄t̄i Dc`rcb Kitj th u P̄t̄i u cvl qv hvq Zv̄t̄K Aur̄f̄ etj | GB Aur̄f̄ t̄K e`envi Kti c̄q̄ct̄Yi gva`tg ga`gv ubYq̄ Kiv hvq | mvi w̄ 4.3.1-G c̄l̄Ē Dc̄v̄Ē e`envi Kti u P̄t̄i 4.3.1-G GK̄u Db Aur̄f̄ A`v̄b Kti ga`gv ubYq̄ Kiv ntj v |



u P̄t̄i 4.3.1: c̄q̄ct̄Yi gva`tg ga`gv ubYq̄ (mvi Yx 4.3.1-Gi Dc̄v̄Ē e`envi Kti)

ga`gvi exRM̄w̄ȲZK `ekó (Algebraic Property of Median)

t̄K`x̄q̄ c̄ĒȲZ̄vi GK̄u Ab`Zg c̄v̄b cwi gvc̄ inm̄t̄e ga`gvi GK̄u M̄w̄ȲZK `ekó i t̄q̄t̄Q | t̄mB `ekó u ntj v, h̄w̄ Avgiv FȲv̄Z̄K u P̄ȳt̄K D̄t̄c̄ q̄v̄ Kivi Z̄t̄e ga`gv t̄`t̄K c̄Z̄u u ḡv̄t̄bi u P̄z̄i thv̄M̄dj̄ u Ab` th t̄Kv̄b gvb t̄`t̄K c̄Z̄u u ḡv̄t̄bi u P̄z̄i thv̄M̄dj̄ Āt̄c̄ q̄v̄ q̄z̄i n̄t̄e | u P̄ȳ D̄t̄c̄ q̄v̄ Kivi uel̄q̄ūt̄K Āb̄t̄c̄ q̄z̄i etj | t̄Kv̄b u P̄z̄i h̄w̄ FȲv̄Z̄K nq̄ t̄mB FȲv̄Z̄K u P̄ȳt̄K āv̄Z̄K inm̄t̄e ai t̄Z̄ n̄t̄e | exRM̄w̄ȲZK f̄it̄e Aur̄f̄ e`v̄r̄u u ntj v,

h̄w̄ Avgiv FȲv̄Z̄K u P̄ȳt̄K
D̄t̄c̄ q̄v̄ Kivi Z̄t̄e ga`gv t̄`t̄K
c̄Z̄u u ḡv̄t̄bi u P̄z̄i
thv̄M̄dj̄ u Ab` th t̄Kv̄b gvb
t̄`t̄K c̄Z̄u u ḡv̄t̄bi u P̄z̄i
thv̄M̄dj̄ Āt̄c̄ q̄v̄ q̄z̄i n̄t̄e |

$$\sum |x_i - ga'gv| = 1722$$

thLvfb, $x_i = DcvEivki gvb$

$$\sum = thvMdj wPy$$

$$| | = Abtc\eta \hat{U} \text{ (absolute bar)}$$

A_w, ga'gv t_tK cZuU gvtbi wPZi Abtc\eta thvMdj Ab'' th tKvb gvb t_tK cZuU gvtbi wPyZi Abtc\eta thvMdj Atc\eta\eta 1722i nte| wciwZfite, Ab'' th tKvb gvb t_tK cZuU gvtbi wPZi Abtc\eta thvMdj ga'gv t_tK cZuU gvtbi wPZi Abtc\eta thvMdj Atc\eta\eta epEi nte| GKiu D`vniYi gra'g weliu tevSvi tPov Kiv hvK| cte©cE D`vniY Abvqx cWRb e'w³i IRb (gvtbi µg Abvqx),

57 69 72 82 86 cvDU

GB DcvEivki ga'gv ntjv 72| GLb Avgiv ga'gvi MwYwZK `enkó'wli h_v_Zv ciw\eta Kti t`Ltev ga'gv (72), 57 Ges 86 msL'v gvb_tjv t_tK cZuU gvtbi Abtc\eta thvMdj tei Kti | (mivw 4.3.3 `be'')

mivw 4.3.3: ga'gvi MwYwZK `enkó'wli h_v_Zv ciw\eta

IRb	cZuU gvb t_tK ga'gvi Abtc\eta wPyZ $x_i - ga'gv$	cZuU gvb t_tK 57-Gi Abtc\eta wPZ $x_i - 57$	cZuU gvb t_tK 86-Gi Abtc\eta wPZ $x_i - 86$
57	$ 57 - 72 = 15$	$ 57 - 57 = 0$	$ 57 - 86 = 29$
69	$ 69 - 72 = 15$	$ 69 - 57 = 12$	$ 69 - 86 = 17$
72	$ 72 - 72 = 0$	$ 82 - 57 = 15$	$ 72 - 86 = 14$
82	$ 82 - 72 = 10$	$ 82 - 57 = 25$	$ 82 - 86 = 4$
86	$ 86 - 72 = 14$	$ 86 - 57 = 29$	$ 86 - 86 = 0$
	54	81	64

Dcti i mivwYZ `úofite j \eta' Kiv hv'Q th,

$$\sum |x_i - ga'gv| = 54$$

$$\sum |x_i - 57| = 81$$

$$\sum |x_i - 86| = 64$$

∴ 54 < 81 Ges 54 < 64

ciƒVĒi gj'iqb

beƒK cĕæ

mĭWK DĒti i ciƒk ĭJK (√) ĭPý ĭ b -

1/ Ūga'gv GKĭU DcvĒi ĭkƒK mgvb fĭtM fĭM Kti |Ō

- K. ƒ fĭtM
- L. ĭZb fĭtM
- M. Pvi fĭtM
- N. cvƒi fĭtM

2/ ĭbƒPi DcvĒi i ĭkgyj vq ga'gv msL'v tKvbĭU?

- 57 69 72 82 86
- K. 69
 - L. 72
 - M. 82
 - N. 86

3/ ga'gv ĭbYƒqi tKvb mƒĭU mĭWK?

- K. $L_1 + \frac{\frac{n}{2} - F}{f} \times C.I.$
- L. $L_1 + \frac{n - F}{f} \times C.I.$
- M. $L_1 + \frac{\frac{n}{2} - f}{F} \times C.I.$
- N. $L_1 + \frac{\frac{n}{2}}{f} \times C.I.$

msĭB cĕæ

1/ ga'gv ĭK?

2/ ga'gvi exRMĭYĭZK %ĭkó' ej tZ ĭK terSvq?

iPbvj-K cĕæ

1/ ga'gv ĭK? ga'gvi mĭear I Amĭeavmgƒ Avtj vPbv Ki ab?

2/ ĭbƒPi DcvĒi e'enri Kti tj LĭPƒĬi gva'tg ga'gv ĭbYƒ Ki ab |
2 4 4 6 8 8 8 9 10 10 12 15 15 16 18

cW - 4

**MwYwZK Mo
Arithmetic Mean**

GB cvW tkfI hv Rvbr hrte —

- MwYwZK Mo wK
- MwYwZK Mo wBYqj mF
- MwYwZK Mtoi exRMwYwZK `enkó`
- msWqjB cxwZtZ MwYwZK Mo wBYq
- web`ÍDcvE t_ tK MwYwZK Mo wBYq
- MwYwZK Mtoi myeav I Amyeav

MwYwZK Mo wK (What is Arithmetic Mean)?

mgvR Mtel Yvq tK`iq cEYZvi th cwigvcwU mefPiq tekx Rbwctj Ges euj e`eüz tmuU ntjv MwYwZK Mo| MwYwZK Mo tK A tK mgq thwRZ Mol ejv ntq _vK| mvari Yfite, cEz w` tbi e`envti Mo kãuU ej tZ hv terSvq cwimsL`wKfite e`eüz MwYwZK Mo-Gi A_ wU ZvB| hLb Avgiv ewj th, tmij g GKrb Mo QvI (average student), ZLb Avgiv GwUB ewS th, tmij g Lg fij djvdj Kti wAvevi Lg Lvivc djvdj I Kti wB| AwKvsk QvI-QvI xiv thgb djvdj Kti tQ tmij gl tZgb Kti tQ| tmij tgi GB Ae`vbwU gj`wqZ nq GKwU meRb `xKZ.gv tbi cwitcE tZ| tmB gvbwU wba wZ nq tmij tgi tkYxi cEz wU QvI-QvI x cir`vq th baf tctq tQ Zvi thwM dj tK tgvU QvI msL`v w` tq fivM Kti| msL`v ZvE Kfite tmB gvbwUB ntjv MwYwZK Mo| Avtiv mybw` e`fite ejv hvq th, GKwU DcvE i wkgvjvi mKj gv tbi thwM dj tK DcvE i wki tgvU msL`v w` tq fivM Kiti thB msL`v gvbwU cvl qv hvq tmuUB ntjv MwYwZK Mo| AZGe, msAv Abjvqx,

mgvR Mtel Yvq tK`iq cEYZvi th cwigvcwU mefPiq tekx Rbwctj Ges euj e`eüz tmuU ntjv MwYwZK Mo| MwYwZK Mo tK A tK mgq thwRZ Mol ejv ntq _vK|

$$\text{MwYwZK Mo} = \frac{\text{DcvE i wkgvjvi mKj gv tbi thwM dj}}{\text{DcvE i wki tgvU msL`v}}$$

MwYwZK Mo mvari YZt `cKv tii ntq _vK – mij MwYwZK Mo (simple arithmetic mean) Ges fih9 MwYwZK Mo (weighted arithmetic mean)| Avgiv Rwb th, DcvE i wkgvjvi mKj gv tbi thwM dj tK DcvE i wki tgvU msL`v w` tq fivM Kiti th msL`v gvbwU cvl qv hvq tmuUB ntjv MwYwZK Mo| GB msAvU mvari Yfite MwYwZK Mtoi cE`qMZ tgsij K A_ tK cji vcvj avi Y Kti Ges GB msAv Abjvqx wBYE MwYwZK Mo tKB mij MwYwZK Mo etj | mij MwYwZK Mo ntjv cEz`wK Rretb e`eüz Mo| GKwU D`vni Y t`qv hvK| aiv hvK, 5wU cwiv tii m`m` msL`v h`vµ t9 4, 3, 2, 5 I 6 Rb| AZGe, msAv Abjvqx,

MwYwZK Mo mvari YZt `cKv tii ntq _vK – mij MwYwZK Mo Ges fih9 MwYwZK Mo|

$$\text{MwYwZK Mo} = \frac{4 + 3 + 2 + 5 + 6}{5} = \frac{20}{5} = 4$$

∴ 5wU cwiv tii Mo AvKvi ntjv 4
A_ tK, 5wU cwiv tii Mto 4 Rb m`m` i tqtQ|

Dcti i D`vni tY gvT 5uU cwiveti i K_v ejv ntqtQ Ges cZuU cwiveti i m`m` msL`vi gvbU gvT GKevi D`j mLZ ntqtQ | uKŠ`hv 50uU cwiveti i DcvE`ubtq AvgvT`i MnyuZK Mo ubYq Ki tZ nq Ges t`Lv hvq th, 3uU cwiveti i m`m` msL`v 2 Rb, 7uU cwiveti i m`m` msL`v 3 Rb, 12uU cwiveti i m`m` msL`v 4 Rb, 18uU cwiveti i m`m` msL`v 5 Rb, Ges 10uU cwiveti i m`m` msL`v 6 Rb, tm t`t`T nel quU uKQyv mgm`v `Zix Kti |

MnyuZK Mo ubYq i Rb` AvgvT`i `uU msL`v gvb Rbvr c`qvRb | GKU ntjv, cwiveti i tgvU msL`v Ges DcvE`i mki mKj gvTbi thvMdj | Avgiv Rvb th, cwiveti i tgvU msL`v ntjv 50 | uKŠ` mKj gvTbi thvMdj u Avgiv Rvb br | tmU AvgvT`i ubYq Ki tZ nte | G t`t`T thvMdj u nte fihv thvMdj | tKbbv, G Rb` cZuU cwiveti i AvKvi tK cwiveti i msL`v w`tq , Y Kti c`B , Ydtj i mgw`tK thvMdj wmvte e`envi Ki tZ nte | AZGe,

$$\begin{aligned} mKj \text{ gvTbi fihv thvMdj} &= (3 \times 2) + (7 \times 3) + (12 \times 4) + (18 \times 5) + (10 \times 6) \\ &= 6 + 21 + 48 + 90 + 60 \\ &= 225 \end{aligned}$$

c`qvRbvx `uU msL`v gvbB Avgiv tctq uMtquQ | Avgiv GLb MnyuZK Mo ubYq Ki tZ cwiv |

$$\begin{aligned} \text{MnyuZK Mo} &= \frac{225}{50} \\ &= 4.5 \end{aligned}$$

AZGe, 50uU cwiveti i Mo AvKvi ntjv 4.5 |

A`P, Avgiv ej tZ cwiv th, 50uU cwiveti i cZuU cwiveti Mto 4.5 Rb Kti m`m` i tqtQ | GB c`x`Z tZ ubYqZ MotK fihv Mo etj | Kvi Y, Mo ubYq i t`t`T cZuU `dvtK (item) h_vh_ , i`Zi (weight) w`tq thvMdj u ubYq Kiv ntqtQ |

MnyuZK Mo ubYq i m`T (Formula for Computing Arithmetic Mean)

tK`tq c`YZvi cwigic wmvte MnyuZK Mtoi Rbuc`Zvi Ab`Zg Kvi Y ntjv, GuU tK m`eavRbKfvte cZx`Ki gva`tg c`Kik Kiv hvq | GB m`eavU RuUj Ges D`Pzi cwiv msL`mbK e`envi i t`t`T MnyuZK Mtoi e`envi tK mnRzi Kti tZvtj | MnyuZK Mo ubYq i exRMnyuZK m`T u ntjv,

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N}$$

MnyuZK Mtoi GB m`T u tK Ab`vteb Ki tZ ntj m`T e`eüZ cZx`K , tjtK e`stZ nte | GLvtb x_i ej tZ 'x' Pj tKi GK GKU gvb, $\sum x_i$ ej tZ 'x' Pj tKi cZuU gvTbi mgw`, N ej tZ DcvE`i mki tgvU msL`v Ges 'x' ej tZ 'x' Pj tKi MotK tevSvq | AvgvT`i D`vni tY 5uU cwiveti i m`m` msL`v ubtq mij MnyuZK Mo ubYq Kiv ntqtQ etj mgw` uP`yi Dcti th 'N' A`ji u tJ Lv ntqtQ tmU tmB 5uU cwiveti Ges i ej tZ GKU cwiveti tK ubt`R Kti |

$$\text{thgb, } \sum_{i=1}^5 x_i |$$

$$A_{\text{f}}, \sum_{i=1}^5 x_i = x_1 + x_2 + x_3 + x_4 + x_5 = 4 + 3 + 2 + 5 + 6 = 20$$

thnZz`N' ejtZ DcvEimki tgvU msL`vK tevSvq, tmtnZzAvgt`i D`vniY Abyvqx
 N = 5 | GB AskwU mfi gta`hy Kijt Avgiv cvB,

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N}$$

Gi A`njv, 'x' Pjtki x_1 t`K iiaKti x_N chsmKj gvbK thM Kitz nte Ges
 tmB thMdj tK N w`tq fM Kitz nte | cZxtKi gva`tg mfuK Gfite tj Lv nq,

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_N}{N}$$

hw`tkvb A`uozv br`vK, Zte Avgiv mfuK Aviv mnRfite Dc`vcb Kitz cwi,

$$\bar{x} = \frac{\sum x}{N}$$

GLb t`K GB mnR Avfe`w`UjtK Avgiv MnyZK Mtoi m`nmte e`envi Kite | Buzgta`
 Avgiv trtbw th, $\sum x = 20$ Ges $N = 5$ | tm t`jt m` Abyvqx,

$$\begin{aligned} \bar{x} &= \frac{20}{5} \\ &= 4 \\ \therefore \bar{x} &= 4 \end{aligned}$$

MnyZK Mo w`yqi GB tgvij K mfuK mij MnyZK Mo Ges Awb`DcvE t`K Mo
 w`yqi m` nmte e`envi Kiv nq | fihy MnyZK Mo w`yqi mfi exRMnyZK
 Avfe`w`U ntv,

$$\bar{x} = \frac{\sum wx}{\sum w} \quad \text{ev} \quad \bar{x} = \frac{\sum wx}{N}$$

thLvtb, $w = \text{cZiU`dvi fi}$
 $x = \text{cZiU`dvi gvb}$
 $\sum wx = \text{cZiU fihy gvtbi mgw}$
 $\bar{x} = \text{fihy MnyZK Mo}$

$$\begin{aligned} \therefore \sum wx &= w_1 x_1 + w_2 x_2 + \dots + w_N x_N \\ \sum w &= w_1 + w_2 + \dots + w_N \\ N &= \text{DcvEimki tgvU msL`v} \end{aligned}$$

MwYwZK Mtoi exRMwYwZK `enkó" (Algebraic Properties of Arithmetic Mean)

MwYwZK Mtoi `gU exRMwYwZK `enkó" itqtQ| cUgwU ntjv, Mo t_tK cUzUw gvtbi wePzuZi thwMdj memqB 000 nte| cUzUw Ki gra_tg GB `enkó"Utk Gfvte cKvk Kiv hvq,

$$\sum_{i=1}^5 (x_i - \bar{x}) = 0$$

mviwY 4.4.1-G MwYwZK Mtoi GB `enkó"U h_v `Zv ciXqVv Kti t`Lv ntqtQ| aiv hvK, Avgiv 5wU msL`vi Mo wBYq Ki tZ PvB| tmB msL`v, tjv ntjv 20, 23, 25, 27 Ges 30| mF` Abgvqx,

$$\begin{aligned} \bar{x} &= \frac{\sum_{i=1}^N x_i}{N} \\ &= \frac{20 + 23 + 25 + 27 + 30}{5} \\ &= \frac{125}{5} \end{aligned}$$

∴ $\bar{x} = 25$

GB wBYq Mo t_tK cUzUw gvb weqvM Kti tmB weqvMdj, tjv thwM Kiti djvdj wU ntqtQ 000| MwYwZK Mtoi gvb Qvor Ab` th tKvb gvb t_tK cUzUw gvb weqvM Kiti Zv 000-Gi tPtq epEi ev qZzi nte| hw` Avgiv 23 msL`wUtk MwYwZK Mo wmwte Abgvb Kwi Ges GB AbvgZ Mo t_tK cUzUw gvbtk weqvM Kti cUzUw gvb, tjv tK thwM Kwi Zte t`Ltev th, thwMdj wU 000 nqvb, ntqtQ 10| hw` AbvgZ Mo 27 nq, tm t`qit thwMdj wU ntqtQ -10|

mviwY 4.4.1: MwYwZK Mtoi cUg `enkó"U [$\sum (x_i - \bar{x}) = 0$] h_v `Zvi cgvY

x_i	$(x_i - 25)$	$(x_i - 23)$	$(x_i - 27)$
20	$(20 - 25) = -5$	$(20 - 23) = -3$	$(20 - 27) = -7$
23	$(23 - 25) = -2$	$(23 - 23) = 0$	$(23 - 27) = -4$
25	$(25 - 25) = 0$	$(25 - 23) = 2$	$(25 - 27) = -2$
27	$(27 - 25) = 2$	$(27 - 23) = 4$	$(27 - 27) = 0$
30	$(30 - 25) = 5$	$(30 - 23) = 7$	$(30 - 27) = 3$
tgvU	0	10	-10

MwYwZK Mtoi wUZiq exRMwYwZK `enkó"U ntjv, Mo t_tK cUzUw gvtbi wePzuZiK eMqKti cUzUw gvb, tjv thwMdj Mo e`wZZ Ab` th tKvb gvb t_tK cUzUw gvtbi wePzuZiK eMqKti cUzUw gvtbi thwMdj Atc`qVv memqB qZzG nte| GB `enkó"i exRMwYwZK Awfe`w`Uw ntjv,

$$\sum_{i=1}^5 (x_i - \bar{x})^2 = 9\check{z}Zg$$

cŭg `enkó'uU cŭvYi tŕŕŕ e`eüZ msL'v gvb, tŕv e`envi Kŕi wŕZxq `enkó'uU h_v_Zv cixŕv Kŕi t`Lv ntŕv mviwY 4.4.2-G| GLvŕb Mo gvb 25 Ges `ŕU AbvgZ Mo gvb (23 | 30) t_ŕK cŭZuU gŕtbi wePzuZi thvMdj wbyŕ Kiv ntqtQ|

mviwY 4.4.2: MmYwZK Mŕoi wŕZxq `enkó'uU [$\sum (x_i - \bar{x})^2 = 9\check{z}Zg$] h_vhZvi cŭvY

x_i	$(x_i - \bar{x})^2$	$(x_i - 23)^2$	$(x_i - 30)^2$
20	$(20 - 25)^2 = 25$	$(20 - 23)^2 = 9$	$(20 - 30)^2 = 100$
23	$(23 - 25)^2 = 4$	$(23 - 23)^2 = 0$	$(23 - 30)^2 = 49$
25	$(25 - 25)^2 = 0$	$(25 - 23)^2 = 4$	$(25 - 30)^2 = 25$
27	$(27 - 25)^2 = 4$	$(27 - 23)^2 = 16$	$(27 - 30)^2 = 9$
30	$(30 - 25)^2 = 25$	$(30 - 23)^2 = 49$	$(30 - 30)^2 = 0$
tŕuU	58	78	183

Dctii mviwYU chŕeŕY Kitj t`Lv hvq th, wŕZxq `enkó'uU cŭvYZ ntqtQ| A_ŕ, Mo t_ŕK cŭZuU gŕtbi wePzuZi thvMdj AbvgZ Mo gvb `ŕU t_ŕK cŭZuU gŕtbi wePzuZi thvMdj Atcŕv ŕŕZZg ntqtQ| Avgv`i cŭB gvb, tŕv ntŕv,

$$\sum (x_i - \bar{x})^2 = 58$$

$$\sum (x_i - 23)^2 = 78$$

$$\sum (x_i - 30)^2 = 183$$

thŕnZz $58 < 78 < 183$

$$tmŕnZz \sum (x_i - \bar{x})^2 = 9\check{z}Zg$$

MmYwZK Mŕoi GB exRMmYwZK `enkó'uUŕK ŕŕZZg eŕMP `enkó'u (least squares property) ejv ntq_vŕK| GB `enkó'uU D`PZI exRMmYwZK e`envŕi i tŕŕŕ MmYwZK MoŕK tek DcŕthvMx Kŕi tZŕj, weŕkl Kŕi cwiŕgZ wePzuZ, mn-m=utŕKŕ mnM, wbfŕv¼, BZ`w wbyŕŕi tŕŕŕ|

msuŕŕB c×uZŕZ MmYwZK Mo wbyŕ (Computing Arithmetic Mean through Short-Cut Method)

cŭZuU `dri thvMdj tei KitZ nq etj cŕ`ŕ c×uZŕZ (direct method) MmYwZK Mo wbyŕ tek mgq I kŕmva` e`vci| ŕm Rb` GB c×uZŕK AŕbK mgq `xN`c×uZI (long method) ejv ntq_vŕK| DcvEi wki msL'v AŕbK tekx ntŕ cŕ`ŕ c×uZi e`envi tek RŕUj ntq_cŕo| dtŕ, mgq Ges kŕy Avi I tekx jŕM| G me mgm'v Govŕbri Rb` cŕivŕ ev msuŕŕB c×uZŕZ (short-cut method) MmYwZK Mo wbyŕ Kiv nq| GB c×uZŕZ DcvEi wki th

tKvb GKilU gvb tK Mo wmwte Abgyv Ki tZ nq Ges D³ AbgyZ ev Kw Z Mo t tK cZiU gvtbi wePvZ ubYq Ki tZ nq| wePvZ wj i thvMdj tK DcvEi mki tgvU msL v w tq fVM Kti cB djvdj tK AbgyZ Mtoi m t thvM Kij th gvbU cvl qv hvq, tmUB ntjv cKZ. MwYvZK Mo| Awb i DcvE t tK msr B c v Z tZ Mo ubYq i m tU ntj v,

$$\bar{x} = A + \frac{\sum_{i=1}^N (x_i - A)}{N}$$

thLv b, A = AbgyZ Mo

x_i = DcvEi mki cZiU gvb

N = DcvEi mki tgvU msL v

(x_i - A) = AbgyZ Mo t tK cZiU gvtbi wePvZ

$\sum (x_i - A)$ = wePvZi thvMdj

$$A_{ev}, \bar{x} = A + \frac{\sum d}{N}$$

thLv b, d = $\sum (x_i - A)$ = AbgyZ Mo t tK cZiU gvtbi wePvZi mgw |

msr B c v Z e'envi Kti 5uU msL vi Mo ubYq Kti wbtgem tU i c qvM t Lv b v nj | aiv hvK, 20, 23, 25, 27 Ges 30 ntj v cE 5uU msL v Ges Gi AbgyZ Mo 25 | m t Abgyv Avgt i 3uU gvtbi c qvRb: N, A Ges $\sum (x_i - A)$ | Avgiv Rvb th, N = 5 Ges A = 23 | GLb AbgyZ Mo t tK cZiU gvtbi wePvZi mgw tei Ki tZ nte | AZGe,

$$\begin{aligned} \sum (x_i - A) &= (20-23) + (23-23) + (25-23) + (27-23) + (30-23) \\ &= (-3) + (+0) + (2) + (4) + (7) \\ &= -3 + 0 + 2 + 4 + 7 \\ &= -3 + 13 \\ &= 10 \end{aligned}$$

Gevti cB gvb t tK m t i gta c qvM Ki tZ nte |

$$\begin{aligned} \bar{x} &= A + \frac{\sum_{i=1}^N (x_i - A)}{N} \\ &= 23 + \frac{10}{5} \\ &= 23 + 2 \\ &= 25 \end{aligned}$$

∴ ubYvZ Mo ntj v 25

cĹ`Ń c×uZ e`envi Kij I Avgiv GKB gvb cřev| thgb,

$$\begin{aligned} \bar{x} &= \frac{\sum x_i}{N} \\ &= \frac{20+23+25+27+30}{5} = \frac{125}{5} = 25 \end{aligned}$$

∴ ubYřZ Mo ntjv 25

web`-DcivĚ t_řK MmYřZK Mo ubYř (Computing Arithmetic Mean from Grouped Data)

cĹ`Ń c×uZ (Direct Method)

Avgř`i cĹ`Ń Rıv cĹ`ŃRb th, web`-DcivĚ `ŃaiřYi ntq _řK – GKıU ntjv newQbřegřıvi (discrete level) Ges AřıKıU ntjv AnewQbřegřıvi (continuous level) | `ŃaiřYi web`řmi třřř cĹ`Ń c×uZi GKB mř-e`envi Kiv ntq _řK | mřıU ntjv,

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

Zře, web`řm `Ńıi třřř cĹ`Ń,tjvi e`vL`vi wřbřv itqřQ| thgb, newQbře web`řmi (discrete distribution) třřř,

- \bar{x} = MmYřZK Mo
- f_i = NUbmsL`v
- x_i = DcivĚi vřki cĹ`ŃıU gvb
- N = DcivĚi vřki tgvU msL`v
- Σ = mğřó řPy

AnewQbře web`řmi (continuous distribution) třřř,

- \bar{x} = MmYřZK Mo
- f_i = cĹ`ŃıU řkYřZ Ařřř NUbmsL`v
- x_i = cĹ`ŃıU řkYři ga`-gvb
- N = NUbmsL`vi mğřó
- Σ = mğřó řPy

newQbře web`řmi t_řK MmYřZK Mo ubYřři Rb` ři hř MmYřZK Mo ubYřři D`vni YıU e`envi Kiv thřZ cřři, hv mvi řY 4.4.3-G Dc`řcZ ntjv|

mviw 4.4.3: 50w cwiwfi i m`m` msL`vi wv`vm

cwiwfi i m`m` msL`v (x_i)	cwiwfi i msL`v (f_i)	($f_i x_i$)
2	3	6
3	7	21
4	12	48
5	18	90
6	10	60
tgw	50	225

mF Abgvqx,

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{225}{50} = 4.5$$

∴ wv`wZ Mo ntjv 4.5

mviw 4.4.4-G cÖ E 100 Rb QvT`i gwmK Li`Pi wnmv`ei wfvE`Z Awew`Obwv`vm t`_tK cÖ`q| c`wZ`Z MwYwZK Mo wv`wZ K`i t` Lv`bv ntjv|

mviw 4.4.4: 100 Rb QvT`i gwmK Li`Pi wv`vm

Li`Pi cw`gvY (C.I.)	QvL msL`v (f_i)	t`kYxi gw`-gvb (x_i)	($f_i x_i$)
1000 – 1500	4	1250	5000
1500 – 2000	13	1750	22750
2000 – 2500	19	2250	42750
2500 – 3000	32	2750	88000
3000 – 3500	20	3250	65000
3500 – 4000	8	3750	30000
4000 – 4500	3	4250	12750
4500 – 5000	1	4750	4750
tgw	100		271000

cÖB gv`b, t`jv mF` cÖqvM Ki`j Avgiv cvB,

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{271000}{100} = 2710$$

∴ wv`wZ Mo ntjv 2710

A`w, 100 Rb QvT`i cÖZ gwm Mo e`q ntjv 2710 UvKv|

Avgir cř`ř c×uZřZ MmYuZK Mo ubYř KiřZ vřtq ř`LřZ cwr0 th, msl`v gıv eo nřj RıUj Zı řeřo hřř`0 Ges mgqř j Mř0 AřK tekř| GřZ řy-řmřřnevi mřebr AřK řeřo hıq| GB mgm`v, řj v Govřbri Rb` Avgrř` i mřvřB c×uZřZ MmYuZK Mo ubYřqi cřqvRb|

msvřB c×uZř (Short-Cut Method)

vew`QbřPj K ev veb`řmi řřřř mřvřB c×uZřZ MmYuZK Mo ubYřqi mřıU nřj v,

$$\bar{x} = A + \frac{\sum f_i(x_i - A)}{N}$$

řhLvřb, \bar{x} = MmYuZK Mo

A = AbřřZ Mo

x_i = Dcvři vřki cřZıU gıv

f_i = NUbmsL`v

N = Dcvři vřki řgvU msl`v

Σ = mgıó řPy

mviřY 4.4.3-G cřř Dcvř Dcvř³ mřř cřqvM Kři MmYuZK Mo ubYř Kiv řřřZ cvři (mviřY 4.4.5 `ře`)|

mviřY 4.4.5: 50ıU cvıvřři m`m` msl`vi veb`řmi řřřřZ mřvřB c×uZřZ MmYuZK Mo ubYř|

x_i	f_i	$(x_i - A)$	$f_i(x_i - A)$
2	3	$(2 - 4) = -2$	-6
3	7	$(3 - 4) = -1$	-7
4 = A	12	$(4 - 4) = 0$	0
5	18	$(5 - 4) = 1$	18
6	10	$(6 - 4) = 2$	20
řgvU	50	0	25

mř Abřřvřř,

$$\begin{aligned} \bar{x} &= A + \frac{\sum f_i(x_i - A)}{N} \\ &= 4 + \frac{25}{50} \\ &= 4 + 0.5 \\ &= 4.5 \end{aligned}$$

\therefore ubYřZ Mo nřj v 4.5

Avew`Qbřvřřmi řřřř mřvřB c×uZřZ MmYuZK Mo ubYřqi mřıU nřj v,

Gm Gm GBP Gj

$$\bar{x} = A + \frac{\sum fd}{N} \times C.I.$$

thLv#b, \bar{x} = MmYwZK Mo

A = AbvgZ Mo

f_i = c#ZwU tkYxi Ašif# NUbmsL`v

d = $\frac{x - A}{C.I.}$ ev A t_#K avc weP#Z

x_i = c#ZwU tkYxi ga`-we`y

C.I. = #kYx e`wB

N = #gvU NUbmsL`v

mvi wY 4.4.6-G c#E 100 Rb Qv#Ti gwmK Li#Pi D`vni YwU e`enri K#i Avgiv Dctiv³ m#FuU gi va`#g msr#B c#vZ#Z MmYwZK Mo wby# Ki #ev|

mvi wY 4.4.6: 100 Rb Qv#Ti gwmK Li#Pi vnm#ei wv#E#Z msr#B c#vZ#Z MmYwZK Mo wby#

#kYx mgv (C.I.)	NUbmsL`v (f_i)	#kYxi ga`-gub (x_i)	avc weP#Z $d = \frac{x_i - A}{C.I.}$	fd
1000 – 1500	4	1250	$\frac{(1250 - 2750)}{500} = -3$	-12
1500 – 2000	13	1750	$\frac{(1750 - 2750)}{500} = -2$	-26
2000 – 2500	19	2250	$\frac{(2250 - 2750)}{500} = -1$	-19
2500 – 3000	32	2750 = A	$\frac{(2750 - 2750)}{500} = 0$	0
3000 – 3500	20	3250	$\frac{(3250 - 2750)}{500} = 1$	20
3500 – 4000	8	3750	$\frac{(3750 - 2750)}{500} = 2$	16
4000 – 4500	3	4250	$\frac{(4250 - 2750)}{500} = 3$	9
4500 – 5000	1	4750	$\frac{(4750 - 2750)}{500} = 4$	4
#gvU	N = 100			$\sum fd = -8$

civēi gj'iqb

be³K cka

mivK Dēti i civk iJK (√) iPy i b –

1/ mgvR MteI Yvq tK>`iq cēYZvi tKiv civi givciU tekI Rbivcē Ges eūj e'ēüZ?

- K. cPžK
- L. ga'gv
- M. MmYuzK Mo
- N. R'ivguzK Mo

2/ 5uU civi evti i m`m` msL'v h_vµtg 4, 3, 2, 5 I 6| G t'it' 5uU civi evti i Mo AvKvi KZ?

- K. 5
- L. 4
- M. 3
- N. 6

3/ mivavi Y MmYuzK Mo iBYēqi mF tKiviuU?

- K. $\frac{DcivE\ i\ vnk\ gvj\ vi\ mKj\ gv\ tbi\ thvMdj}{DcivE\ i\ vnk\ i\ tgvU\ msL'v}$
- L. $\frac{DcivE\ i\ vnk\ gvj\ vi\ mKj\ gv\ tbi\ _Ydj}{DcivE\ i\ vnk\ i\ tgvU\ msL'v}$
- M. $\frac{DcivE\ i\ vnk\ gvj\ vi\ mKj\ gv\ tbi\ fivMdj}{DcivE\ i\ vnk\ i\ tgvU\ msL'v}$
- N. Dcti i tKiviuUB bq|

msivfjB cka

- 1/ MmYuzK Mo vK?
- 2/ MmYuzK Mo iBYēqi cē'q| c×iZiuU ve'kdi Y Ki ab|

iPbvji-K cka

- 1/ MmYuzK Mo vK? MmYuzK Mtoi miveav I Amiveavmgn- Avtj vPbv Ki ab?
- 2/ MmYuzK Mo iBYēqi msivfjB c×iZ e'ēni Kti iBtar³ DcivE t_tK MmYuzK Mo iBYē Ki ab|

civēti i m`m` msL'v (x _i)	civēti i msL'v (f _i)
2	3
3	7
4	12
5	18
6	10

tK>`ıq cEYZvi cwi gvc mgfai gta` Zjbr Ges ıberPb
Comparison and Choice between the Measures of Central Tendency

GB cıW tktı hv Rıvr hvte —

- cPžK, ga`gv I MmYıZK Mtoi gta` Zjbr
- tK>`ıq cEYZvi h_vh_ cwi gvc ıberPb

cPžK, ga`gv I MmYıZK Mtoi gta` Zjbr (Comparison between Mode, Median and Arithmetic Mean)

tK>`ıq cEYZvi newfbre cwi gıtcı msÁv, `enkó` Ges ıbYq cııZ Avgıv cteP cıV, tj vtZ Avtj vPbv KtııQ | GLb Avgıv tm mKj cwi gıtcı `enkó`i gta` ıgj, Augj Ges mıgve×Zvi ıfııÉtZ cwi gvc, tj vtK Zjbr Ktı t`Lter |

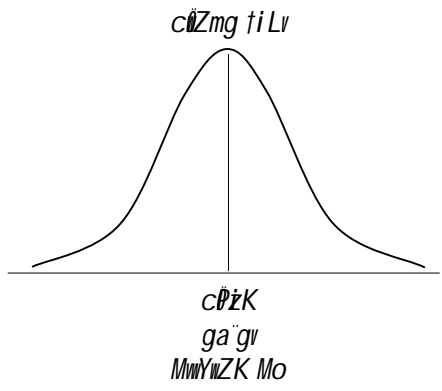
cL`qmZ Abb`Zv (Conceptual Uniqueness)

tK>`ıq cEYZvi cıZıU cwi gıtcı ıbR`ıKQyAbb` `enkó` i tqtQ | thgb, cPžK ntj v Ggb GKıU gvb hv meıPtq tekx NUbmsL`ıK cıZııwaZı Ktı, ga`gv ntj v Ggb GKıU gvb hv GKıU ıeb`ıtmı Ašfı AvqZbtk mgvb `ıfııM fıM Ktı Ges MmYıZK Mo ntj v Ggb GKıU gvb hv GKıU ıeb`ıtmı fıv mıg` i fıv Ktı | ıZııU cwi gıtcı gta` MmYıZK Mo meıPtq tekx e`eüZ Ges Rbııı | ga`gv ıKQy Kg cwi ıPZ Ges mnRZı avı Yıı Dcı cıZııZ | cPžK Avtıv Kg cwi ıPZ GKıU cwi gvc Ges meıPtq mıj Zi avı Yıı Dcı cıZııZ |

eıWgZv (Skewness)

GKıU mıg ıeb`ıtm (symmetrical distribution) cPjK, ga`gv I MmYıZK Mtoi gvb hııcrfıte GK`ıtb ıgtj hvq | A`ı, hıı Avgıv`ı nıvZ GKıU mıg MYmsL`v fıLv (symmetrical frequency curve) `ıK Zte Avgıv t`Lter th, fıLvı tK>`ııZ NUbmsL`ıv meıPtq tekx NbZı i tqtQ hv ıeb`ııııı tK>`ıe`ıııııte cıZfıv nıvZ Ges cPžK, ga`gv Ges MmYıZK Mo GKB msL`v gıtb cKııKZ nıvZ (ıPı 4.5.1 `be`ı) |

tK>`ıq cEYZvi cıZıU cwi gıtcı ıbR`ıKQyAbb` `enkó` i tqtQ | thgb, cPžK ntj v Ggb GKıU gvb hv meıPtq tekx NUbmsL`ıK cıZııwaZı Ktı; ga`gv ntj v Ggb GKıU gvb hv GKıU ıeb`ıtmı Ašfı AvqZbtk mgvb `ıfııM fıM Ktı Ges MmYıZK Mo ntj v Ggb GKıU gvb hv GKıU ıeb`ıtmı fıv mıg` i fıv Ktı | ıZııU cwi gıtcı gta` MmYıZK Mo meıPtq tekx e`eüZ Ges Rbııı |



ıPı 4.5.1: mıg MYmsL`v fıLvı cPžK, ga`gv I MmYıZK Mtoi Ae`ıı

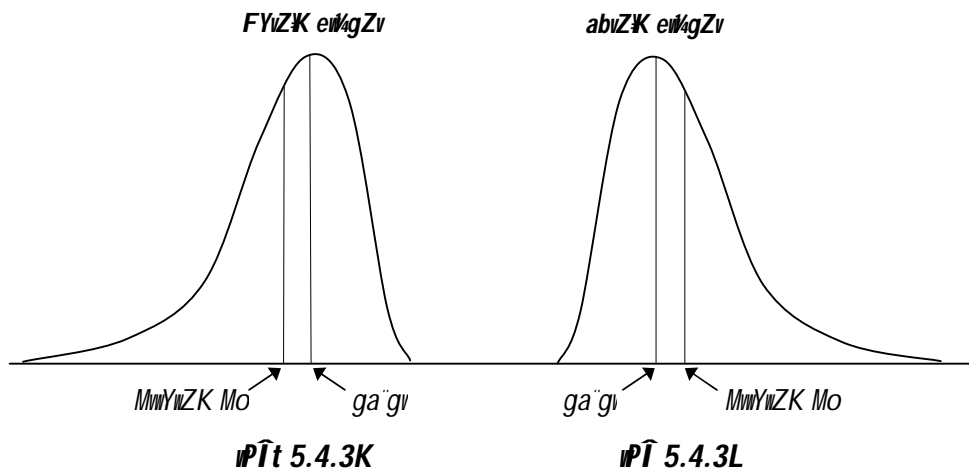
ewig webvfm cPzK, ga'gv I MwyZK Mtoi gvb wfbentq vtK | hw` GKwU myg ti Lvi GKwU cbsiewaz ntq ewig ifc tbq, tm tqtT cPzKi gvtbi tKvb cwieZB nq bv, wKS' ga'gvi gvb cwiewZ ntq ewig cttshi w` tK Ae`vb MhY Kti | hw` webvfmU avZKfite ewig nq Zte ga'gvi gvb ewx crq Ges MwyZK Mtoi gvb ewx crq Avi I tekx | KviY, cPzKi GK w` tK i'ayth wKQyAwZwi 3 NUbmsL`vi mgvtek NtU ZvB bq, GB NUbmsL`v, tjv cPzK t` tK wePzZi cwigvYI ewotq t` q | ZLb cPzK, ga'gv I MwyZK Mtoi m`uKwU `vcvq wbgdfc,

$$MwyZK Mo > ga'gv > cPzK$$

hw` webvfmU FYvZKfite ewig nq Zte cPzKi gvb nq mtePP Ges MwyZK Mtoi gvb nq mebgdf A`f,

$$MwyZK Mo < ga'gv < cPzK$$

wP` 5.4.3K Ges 5.4.3L-G m`uKwU c` k` Kiv nj |



MwyZK I eRMwyZK `enkó" (Mathematical and Algebraic Properties)

hw` DcvEivki mKj gvtbi mgwó, DcvEivki tgvU mSL`v Ges MwyZK Mtoi gta` th tKvb `w` gvb Rvb vtK Zte ZZxq gvbuU wYq Kiv m`e |

MwyZK Mo eRMwyZK cwieMYvbi Rb` AwakZi Dc`hwMx | thtnZz $\bar{x} = \frac{\sum x_i}{N}$, tmtnZz

hw` DcvEivki mKj gvtbi mgwó, DcvEivki tgvU mSL`v Ges MwyZK Mtoi gta` th tKvb `w` gvb Rvb vtK Zte ZZxq gvbuU wYq Kiv m`e | A`f,

$$\bar{x} = \frac{\sum x_i}{N}; \quad \sum x_i = N\bar{x}; \quad N = \frac{\sum x_i}{\bar{x}}$$

Dch` fi e`envi Kti KZ, tjv MwyZK Mtoi m`sj Z Mo wYq Kiv thtZ cvti, hv cPzK I ga'gvi tqtT m`e bq | MwyZK Mtoi `w` , iazcYqMwyZK `enkó" itqt0,

$$\sum (x_i - \bar{x}) = 0$$

Ges $\sum (x_i - \bar{x})^2 = \text{qzZg}$

u0Ziq `enkó`Uji Kvi tY MmYwZK Mo tK we`iZi cwi gvtci wfiE` inmvte MY` Kiv nq| MmYwZK Mo D`PZi cwi msL`vb (thgb, mn-m=ú tKp mnM, ubfPv, cwi ugZ ti Lv, BZ`w`) ubYŷqi tŷŷŷ GKwU , iazcY`A t cŷŷK (function) inmvte Kiv Kti| thnZzga`gv t` tK c0ZuU gvtbi wePzZi thvMdj l ŷŷZg, tmnZze`iZi uKQycwi gvc ga`gvi Dci wfiE` Kti l ubYŷZ nq| thgb, ga`gvi wePzZ| uKŠ`c0Z tKi tŷŷŷ tZgb tKvb exRMmYwZK cwi MYbv m`e bq|

tkYxe×KiY (Classification)

Aweb`-l DcvE`ivk, mivRv t v i mkgjv Ges MYmsL`v ubtekb t` tK MmYwZK Mo ubYŷ Kiv hvq| hLb MYmsL`v ubtekb t` tK MmYwZK Mo ubYŷ Kiv nq, ZLb ubYŷZ gvbuU Aweb`-l DcvE`ivk t` tK MmYwZK Mtoi ubYŷZ gvtbi Lŷ KivQvKwQ nq| web`vnuU hZ mŷg nte, GB gvb`ŷu ZZ KivQvKwQ nte| A`ŷ, MmYwZK Mo ubYŷqi tŷŷŷ DcvE`i tkYxe×KiYi c0qvRbxqZv Ri a i x b q|

uKŠ` ga`gv ubYŷqi tŷŷŷ DcvE`ivki tkYxe×KiY c0qvRbxq| ga`gv ubYŷqi Rb` AŠZt tK` iŷ gvb , t j v tK mivRtq ubtZ nq| MYmsL`v ubtekb t` tK ubYŷZ ga`gvi gvb mivRtq tbqv (arrayed) DcvE`ivk t` tK ubYŷZ gvtbi KivQvKwQ nte cŷi h` `dv , t j v i web`v m (distribution of items) ga`gv m`ŷj Z tkYxi gta` ubqwgZfvte Ae`vb Kti |

MYmsL`v ubtektbi ga` t` tK c0Z tKi Ae`vb Lŷ mn tR ubYŷ Kiv hvq| Zte mivRv Yfvte mivRv DcvE`ivk t` tK c0Z tKi Ae`vb ubYŷ uKQyv Kómv`| thgb, RbmsL`vi wfiE` tZ mivRv evsj v` tki tRj v , t j v t` tK tKvb c0Z tK ubYŷ m`e bq| h` tm , t j v tK RbmsL`vi AvKv t i i wfiE` tZ tKvb tkY tZ mivRv hvq, GKgv l ZLb tKej c0Z tKi c0YZnuU j ŷ` Kiv th tZ cŷi |

Amg tkYx e`vB (Unequal Class Intervals)

wevfbetkYx e`vB tZ web`-l DcvE`i tŷŷŷ MmYwZK Mo ubYŷ Kiv hvq| Zte ubYŷZ MmYwZK Mtoi gvbuU mgvb tkYxe`vB m`ú bweb`v m ev Aweb`-l DcvE`ivk t` tK ubYŷZ gvb t` tK wfbet nte v tK| hLb web`vnuU chŷŷ cwi gvtY evŷg nq ZLb tkYx e`vBi wfbzv ev Amg tkYx e`vBi Dcv`wZ N tU| uKŠ` Amg tkYx e`vB m`ŷj Z MYmsL`v ubtekb t` tK Lŷ m tŠ l RbKfvteB ga`gv ubYŷ Kiv m`e| G ai tYi cwi w`wZ tZ c0Z tK ubYŷ Kiv hvq h` c0Z tK m`ŷj Z tkYx e`vB Ges Gi cteŷ cŷi i tkYx e`vB nuU mgvb nq|

Dbŷŷ tkYx mŷgv (Classes with Open End)

GKwU web`v tmi GK c0Š l hLb 0 tKvb GKwU msL`vi tPtq Kg0 (thgb, 10 -Gi tPtq Kg) ev Ab` c0Š l 0 tKvb GKwU msL`vi tPtq tekw0 (thgb, 80-Gi Dtaŷ G ai tYi tkYx mŷgv v tK, ZLb G ai tYi MYmsL`v ubtekb f j MmYwZK Mtoi Rb t` q| Kvi Y, G ai tYi tkYx mŷgv t` tK tkYxi ga`-gvbuU (mid-value) m tŠ l RbKfvte ubYŷ Kiv m`e nq bv| uKŠ` ga`gv ubYŷqi tŷŷŷ Dbŷŷ tkYx mŷgv tKvb m gm`v b q| Dbŷŷ tkYx c0Z tKi gvb ubYŷqi c0Lqv tK l R wU j Zi Kti tZv t j bv| KLb l KLb l Avgiv hLb gvi vZKfvte evŷg web`v m ubtq Kiv R Kw i, ZLb c0Z tKi gvb nq web`v tmi tkl c0Š l ev tkl c0Š l KivQvKwQ Ae`vb Kti | G ai tYi cwi w`wZ tZ c0Z tK ubYŷqi tKvb thŠ v tZ v v tK bv| Kvi Y, G ai tYi web`v tmi tŷŷŷ c0YK tK` iŷ c0YZvi c0Z nuwazKvix cwi gvc b q|

cŷŷ-K gvtbi Dcv`wZ c0Z tK tKvbP fvteB cŷŷeZ Kti bv Zte ga`gv mŷgv` uKQyv cŷŷeZ nq|

Pig gub (Extreme Values)

hLb GKIU DcvEiviki gta" iKQyPig gubbi Dcv-wZ _vK, ZLb MnywZK Mtoi gvbU gviZKfite cfwEZ nq| GKIU gvT Pig gubbi (tQvU ev eo) Dcv-wZl weawSKi MnywZK Mo gubbi Rbfr tZ cvti | Pig gubbi Dcv-wZ cPzKtK tKvb fiteB cfwEZ Kti br Zte ga'gv mvgvb" iKQy cfwEZ nq| GKIU D`vniYi gva'tg melquUi , iaz; Abgweb Kiv hvK| aiv hvK, Avgv' i nvtZ wbgij mLZ mvZiU msL`v gvb i'tqtQ,

10 14 15 15 17 18 20

GB gvb,tjvi MnywZK Mo ntjv 15.57, ga'gv 15 Ges cPzK ntjv 15| hw` Dctiv³ i vkgvj vi mvt_ Aog gvb wmwte GKIU Pig gub 30 thvM Kiv nq, Zte MnywZK Mtoi gvb nte 17.37, iKs' ga'gv nte 16| iKs' cPzKti gub Acwi ewZ' _vKte| hw` Aog Pig gub wmwte Avtiv GKIU epEi gvb 150 hY Kiv nq, Zte MnywZK Mo nte 32.37 Ges ga'gv I cPzK cteP gubB Acwi ewZ' _vKte (A_#r, h_vmtg 16 Ges 15)|

Pig gub m'oj Z GKIU web'vmtK eYvvi t'jtT MnywZK Mo ntjv GKIU weawSgj-K msL`v gvb| cKZct'j, mvgwRK ev`eZvq Ggb iKQyPjK i'tqtQ hv cKwZMzfiteB Pig gub m'oj Z| thgb, Avtqi t'jtT j'j' Kiv hvq th, Awakusk gvb'j i Avq KivQvKwQ _vKtj I iKQy gvb'j i Avq A`fwieKfite tekx ev Kg nq _vK| G ai'tYi Pj'tKi t'jtT MnywZK Mo tKvb Av`k'ev cZubwaz'kj cwi gvc n'tZ cvti br| G t'jtT GKIU DEg cwigvc wmwte ga'gvi e'envi metPq DcthvMx| wetkl wetkl cwi w`wZtZ cPzKI e'envi Kiv th'tZ cvti, iKs' tKvb fiteB MnywZK Mo bq|

tK`iq cEYZvi h_vh_ cwigvc ubePb (Selection of Appropriate Measure of Central Tendency)

Ab'vb" cwigvci Z'zbvq MnywZK Mo bgbv wePzZi Oviv metPq Kg cfwEZ nq| iKs' hw` web'vmtU ew'g nq, tm t'jtT MnywZK Mo tKvb fiteB tK`iq cEYZvi Av`k'ev cwigvc wmwte wetePZ n'tZ cvti br|

DcvEiviki cKwZtK eYvV Ki'tZ wMtq GKRb cwimsL`vbie` eYvvi tKskj wmwte tKvb cwigvcU e'envi Ki'teb tm wltq wmxvS' MhY GKIU mgm`v `Zix Kti| tK`iq cEYZvi cwigvc ubePbi melquU wbfP Kti - cUgZt, DcvEiviki cKwZi Dci; wDZxqZt, MtelYvi melqe`i Dci; ZZxqZt, DcvE w'kt'it'Yi cKwZi Dci; Ges PZzZt, tK`iq cEYZvi cE`quUi Dci hv GKIU m'jv' 0 D't'k" e'envi Kiv n'tq _vK|

hw` web'vmtU m'g nq Zte wZbU cwigvci th tKvb GKIU e'envi Ki'tjB Ptj| Zte MnywZK Mtoi e'envi memqB c'vb wetePbvq _vKv DvPr| KviY, Ab'vb" cwigvci Z'zbvq MnywZK Mo bgbv wePzZi Oviv metPq Kg cfwEZ nq| iKs' hw` web'vmtU ew'g nq, tm t'jtT MnywZK Mo tKvb fiteB tK`iq cEYZvi Av`k'ev cwigvc wmwte wetePZ n'tZ cvti br| cPzK ev ga'gvi e'envi G t'jtT tkq| th web'vmt AwZwi³ cwigvY wePzZ ev AmgmE'Zv (heterogeneity) _vK, tm t'jtT ga'gvB ntjv tkb cwigvc|

GKIU tMvxi gta" GKRb e'w³i Avtci'jK Ae`vb wY'q'j t'jtT tK`iq cEYZvi cwigvc wmwte ga'gvi ubePbiU nte h_vh_ | thgb, cix'jvi djvdj cKwKZ nevi ci t`Lv tMj th, iwdKmn K'k'ki 20 kZvsk OvT-OvT'x 'A' tMw tctq'Q Ges mtevP b'af D'tv'Q 97| iwdK 'A' tMw tctj I Ab'vb' i Z'zbvq Zvi Ae`vbU t'gai'v'tgi tKvb Ae`v'v' i'tqtQ tm Zv Rvb'tZ P'q| GB Avtci'jK Ae`vb (relative position) wY'q'j Rb" ga'gvi e'envi nte tK`iq cEYZvi h_vh_ cwigvc| PZzR, `knwiK ev kZnwiK e'envi Kti GB me Ae`vbgj-K eYvV t`qv hvq|

mtev⁹ cwi gvtY NtU hvI qv GKuU mel tqi eY⁹ t`evi Rb` cPžtKi e`envi c⁹qvRb nq| hw` tkD fweI`⁹Yx KižZ AvM⁹x nb Zte cPžK ntjv tk⁹ gvb hv fweI`⁹Yx Kivi Rb` wbf⁹ gvb tei KižZ m⁹nivh` Kti| KviY, Ab`vb` th tKivb gvtbi Zžbvq cPžK ntjv GKuU GKK gvb thLv⁹tb metP⁹tq tekx NUbvi mgv⁹nvi NtU|

G mKj wekI wekI cwi w`wZ Ovov Ab` mKj t⁹q⁹t⁹ MmYwZK Mo ntjv tk⁹`xq c⁹YZvi h_vh_ cwi gvc| Ggb wK ga`gv ev cPžtKi e`envi ht_vch⁹ nI qv m⁹t⁹E⁹l MmYwZK Mtoi e`envi c⁹qvRbvq ntZ cvti| thgb, GKuU web`vmtK cwi gZ tiLvq (normal curve) Dc`vcb KižZ ntj MmYwZK Mtoi e`envi c⁹qvRb| tk⁹`xq c⁹YZvi cwi gvc tK cieZ⁹ cwi mSL`vbK thšw³KZvq e`envi KižZ ntj MmYwZK Mtoi MmYwZK I exRMmYwZK `ewkó mg⁹ G⁹u⁹tK Ab`vb` cwi gvtci Zžbvq tk⁹žjw` tq_vtK|

AwaKusk t⁹q⁹t⁹ tgvU gvtbi (total value) e`envi L⁹ , iazcY⁹ntq`vovq Ges Avgiv Rvnb th, GB tgvU gvtbi e`envi GKgv⁹ MmYwZK Mo wby⁹qi t⁹q⁹t⁹B e`eüZ ntq_vtK| tm KvižY AwaKusk t⁹q⁹t⁹ MmYwZK Mo tk⁹`xq c⁹YZvi tk⁹ cwi gvc w⁹m⁹vte wete⁹wPZ ntq_vtK| cPžK I ga`gv wby⁹qi t⁹q⁹t⁹ DcvE⁹ivki mKj gvb tK e`envi Kiv nq bv|

msw⁹q⁹Bfvte Avgiv ejžZ cwi th, tk⁹`xq c⁹YZvi cwi gvc wbe⁹w⁹tbi t⁹q⁹t⁹ hLb DcvE⁹ivk bvgmPK gv⁹lvq cwi gvcKZ.nq, ZLb cPžK ntjv tk⁹`xq c⁹YZvi GKgv⁹ cwi gvc| hLb GKuU web`v⁹tm`⁹ ev ZtZv⁹ak cPžtKi mgv⁹tek NtU ZLb c⁹ZwU cPžK m⁹wj Z ch⁹fe⁹q⁹Y eY⁹vi Rb` cPžK ntjv tk⁹`xq c⁹YZvi h_vh_ cwi gvc| Zte Aw⁹eb`⁹I Aw⁹ew⁹QbeDcvE` A_ev`⁹ f mSL`K DcvE⁹i t⁹q⁹t⁹ (thLv⁹tb c⁹ZwU ch⁹fe⁹q⁹Y wfb⁹žv c⁹ok⁹ Kti) cPžtKi e`envi h_vh_ bq|

hLb DcvE⁹ivk μgmPK gv⁹lvq cwi gvcKZ.nq ZLb ga`gv ntjv tk⁹`xq c⁹YZvi tk⁹ cwi gvc| Pig gvb ⁹oviv ga`gv c⁹f⁹w⁹eZ nq bv etj ew⁹g web`v⁹mi t⁹q⁹t⁹ ga`gvi e`envi h_vh_ nq| e`w⁹Bgj-K gv⁹lvq cwi gvcKZ.Pj tKi t⁹q⁹t⁹ MmYwZK Mo wby⁹ ntjv h_vh_| DcvE⁹ivki c⁹ZwU gvb e`eüZ nq etj MmYwZK Mo metP⁹tq c⁹Zw⁹baZk⁹xj cwi gvc w⁹m⁹vte wete⁹wPZ nq| wKš`Pig gvb ⁹oviv c⁹f⁹w⁹eZ nq etj G aižYi web`v⁹mi t⁹q⁹t⁹ tk⁹`xq c⁹YZvi Av`k⁹cwi gvc w⁹m⁹vte MmYwZK Mtoi wbe⁹w⁹ m⁹wK bq| hw` wZbwU cwi gvtci gvtbi gta` L⁹ tekx wfb⁹žv t`Lv hvq, Zte tm t⁹q⁹t⁹ wZbwU cwi gvc tKB wby⁹ Kti Dc`vcb Kiv ew⁹Abxq|

hw` wZbwU cwi gvtci gvtbi gta` L⁹ tekx wfb⁹žv t`Lv hvq, Zte tm t⁹q⁹t⁹ wZbwU cwi gvc tPK wby⁹ Kti Dc`vcb Kiv ew⁹Abxq|

mviusk

cPžK ntjv Ggb GKuU gvb hv metP⁹tq tekx NUbmsL`vtK c⁹Zw⁹baZj Kti| Ab`w` tK ga`gv ntjv Ggb GKuU gvb hv GKuU web`v⁹mi Ašf⁹ AvqZb⁹tK mgvb`⁹ f⁹v⁹im f⁹vM Kti Ges MmYwZK Mo ntjv Ggb GKuU gvb hv GKuU web`v⁹mi f⁹iv⁹mg` i⁹q⁹ Kti| wZbwU cwi gvtci gta` MmYwZK Mo metP⁹tq tekx e`eüZ nq| wZbwU cwi gvtci t⁹q⁹t⁹B a⁹ms⁹l Gov⁹tvri Rb` M⁹telK⁹tK th w⁹lq⁹ w⁹i c⁹ž j`⁹ i⁹vlžZ nte tm⁹ w⁹ nt⁹Q: DcvE⁹i tkY⁹x⁹KiY, tkY⁹x m⁹xgv Ges Pig gvb| gvtbi gta` L⁹ tekx wfb⁹žv t`Lv w` žj wZbwU cwi gvc tKB wby⁹ Kti Dc`vcb Kiv ew⁹Abxq|

cıfVİi gj'ıqb

be@K cka

mıWK DEti i cıfk ıJK (√) ıPy ı b -

1| GKıU mıyg web'ıtm cPzK, ga'gv I MmYıZK Mtoi gvb hıcrfıte ıgtj hıq|

- K. GKB ııfb
- L. ıGU cıK ııfb
- M. ıZbıU cıK ııfb
- N. PııU cıK ııfb

2| nıjv Ggb GKıU gvb hv GKıU web'ıtmı fvi mıg' i ııv Kti |

- K. cPzK
- L. ga'gv
- M. R'ııııZK Mo
- N. MmYıZK Mo

3| tK>ıq cEYZvi tKıv cıı gııU exRMmıYıZK cıı MYbri Rb' AıaKZi DcıhıMıx?

- K. cPzK
- L. ga'gv
- M. MmYıZK Mo
- N. Zi 1/2 Mo

msııB cka

1| eıııgZı ıK?

2| ıkyıe x KiY ıK?

iPıııı-K cka

1| cPzK, ga'gv Ges MmYıZK Mtoi gıa' Zııııııı-K Avıj vPbv Ki ab|

2| ıKfıte tK>ıq cEYZvi h_ıh_ cıı gııc ıııııııı Ki v hıq Zı Avıj vPbv Ki ab|